100 points. Show all work to receive full credit. You may use a calculator. CHECK YOUR WORK!!!!

1. (20 pts) Given the following functions $f(x)$, $g(x)$, and $h(x)$ as defined by function, graph, and table respectively.

$$f(x) = \sqrt{32 - 2x^2}$$

$g(x)$
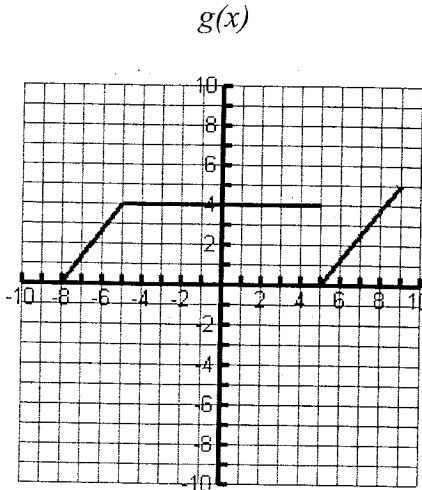
h(x) =	
x	**y**
0	-2
-4	6
5	9
1	-2

Find the following:

A. Domain of $f(x)$

$32 - 2x^2 \geq 0$ $[-4, 4]$
$32 = 2x^2$
$x^2 = 16$ $x = \pm 4$

B. Range of $g(x)$

$[0, 5]$

C. $f(-3)$

$\sqrt{32 - 2(-3)^2} = \sqrt{14}$
≈ 3.74

D. $g(-3)$

4

E. $(f \circ h)(0)$

$f(h(0)) = f(-2)$
$= \sqrt{32 - 2(-2)^2}$
$= \sqrt{32 - 8} = \sqrt{24}$
$= 2\sqrt{6} \approx 4.90$

F. $f(a+h)$

$\sqrt{32 - 2(a+h)^2}$
$= \sqrt{32 - 2(a^2 + 2ah + h^2)}$
$= \sqrt{32 - 2a^2 - 4ah - 2h^2}$

G. $(h+g)(-4) = h(-4) + g(-4)$
$= 6 + 4 = 10$

H. $h(g(h(5))) = h(g(9)) = h(5) = 9$

I. If $g(x)$ is a function, is it one-to-one? Why or why not?

NO. DOESN'T PASS HORIZONTAL LINE TEST

J. The average rate of change of $h(x)$, from $x = -4$ to $x = 5$.

$Ave = \dfrac{h(5) - h(-4)}{5 - (-4)} = \dfrac{9 - 6}{9}$
$= \dfrac{3}{9} = \dfrac{1}{3}$

2. (10 pts) Let $f(x) = \begin{cases} 1-x^2, x \le 0 \\ 2x+1, x > 0 \end{cases}$
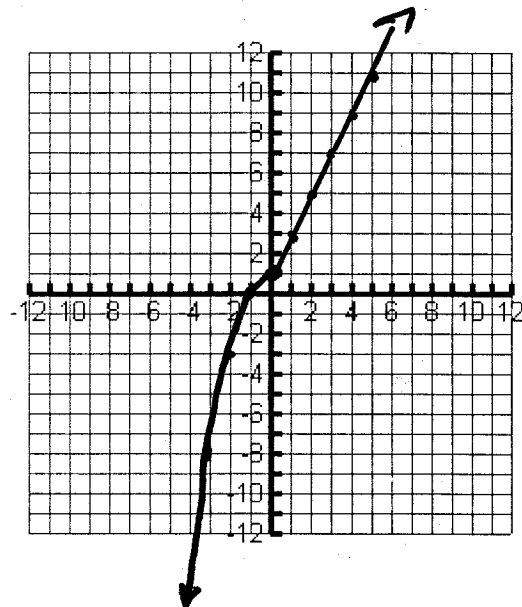
A. $f(-3) =$

$1 - (-3)^2 = 1 - 9 =$

$\boxed{-8}$

B. $f(5) =$

$f(5) = 2(5) + 1 = 11$

C. Graph the function

3. (9 pts) The electrical resistance R of a wire varies directly as its length L and inversely as the square of its diameter d.

A. Write an equation that expresses this joint variation.

$R = \dfrac{KL}{d^2}$

B. Find the constant of proportionality if a wire 1.2 m long and 0.005 m in diameter has a resistance of 200 ohms.

$K = \dfrac{Rd^2}{L} = \dfrac{200(0.005)^2}{1.2} = 0.004167$

C. Find the resistance of a wire made of the same material that is 5 m long and has a diameter of 0.012 m.

$R = \dfrac{0.004167 L}{d^2} = \dfrac{0.004167(5)}{(0.012)^2} = \boxed{144.68 \, \Omega}$

4. (8 pts) Given the following function: $f(x) = \dfrac{2x+1}{3}$. Find the inverse of the function.

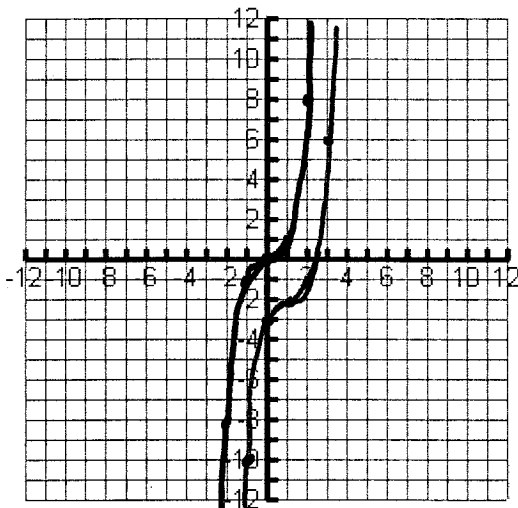$$y = \frac{2x+1}{3} \Rightarrow x = \frac{2y+1}{3}$$

$$3x = 2y+1$$
$$\frac{-1 \qquad -1}{}$$
$$\frac{3x-1}{2} = \frac{2y}{2}$$

$$y = \frac{3x-1}{2}$$

$$\boxed{f^{-1}(x) = \frac{3x-1}{2}}$$

$$1.5x - 0.5$$

5. (9 pts) Sketch the graph of the function $f(x) = x^3$. Describe the transformations necessary to graph the function $g(x) = (x-1)^3 - 2$. Use the information to graph $g(x)$.

$x-1 \Rightarrow$ SHIFT RIGHT 1 UNIT

$-2 \Rightarrow$ SHIFT DOWN 2 UNITS

6. (27 pts) Given the following function: $h(x) = 2x^2 - 8x - 1$. (Round your answers to 2 decimal places if necessary)

What is the domain? $x \in \mathbb{R}$'s $(-\infty, \infty)$

What is the range? $y \geq -9$ $[-9, \infty)$

3 each

What is the vertex? (coordinates)

$(2, -9)$

What is the axis of symmetry? $x = 2$

Find the x-intercepts and y-intercepts. (coordinates)

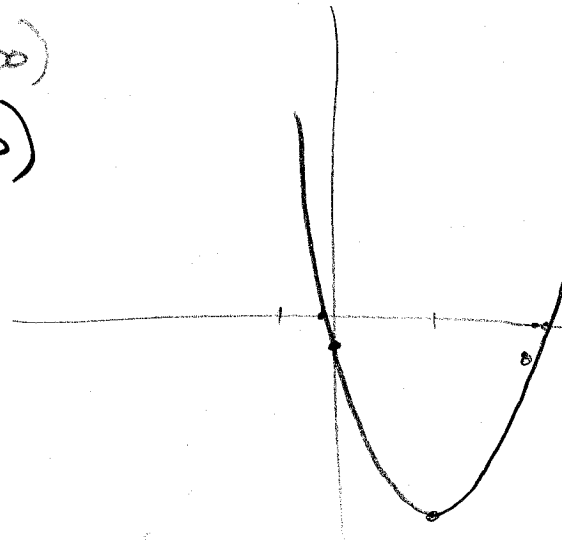X-INTS: $y = 0$ $2x^2 - 8x - 1 = 0$

USING QUADRATIC FORMULA

$$x = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(-1)(2)}}{2(2)} = \frac{8 \pm \sqrt{64+8}}{4} = \frac{8 \pm \sqrt{72}}{4}$$

$$= \frac{4 \pm 3\sqrt{2}}{2}$$

$\left(\frac{4-\sqrt{3}}{2}, 0\right)$ & $\left(\frac{4+\sqrt{3}}{2}, 0\right)$

$\approx$ $(-0.12, 0)$ & $(4.12, 0)$

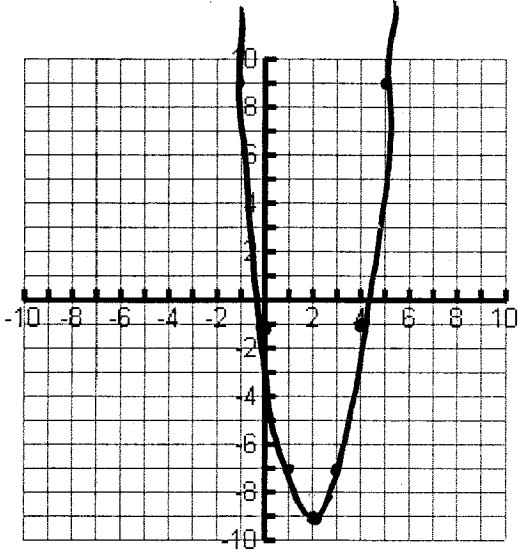Y-INT: $x = 0$
$y = -1$
$(0, -1)$

Does the graph have a minimum or maximum value? What are the coordinates of the maximum or minimum?   MINIMUM $x = \frac{-b}{2a} = \frac{-(-8)}{2(2)} = \frac{8}{4} = 2$

$h(2) = 2(2)^2 - 8(2) - 1 = -9$ $(2, -9)$

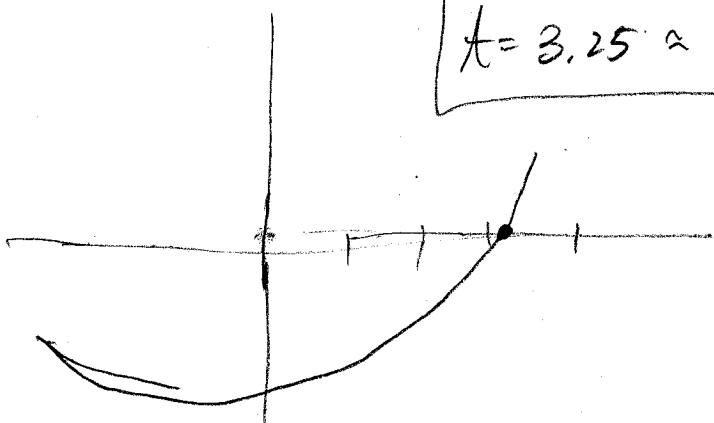Graph the function. Include a table of points used to construct your graph.

x	h(x)
0	-1
2	-9
1	-7
3	-7
4	-1
5	9
-1	9

7. (8 pts) A ball is thrown downward from a tall building. The distance traveled by the ball in t seconds is $d(t) = 16t^2 + 32t$, where d is in feet. How long (to the nearest tenth) will it take the ball to fall 274 ft?

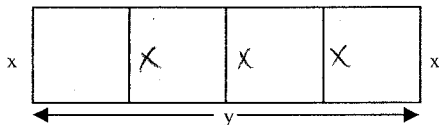$$16t^2 + 32t = 274 \qquad 16t^2 + 32t - 274 = 0$$

GRAPH:

$$\boxed{t = 3.25 \approx 3.3 \text{ sec}}$$

QUADRATIC FORMULA:

$$t = \frac{-32 \pm \sqrt{32^2 - 4(16)(-274)}}{2(16)}$$

$$= \frac{-32 \pm \sqrt{18560}}{32}$$

$$= \frac{-1 \pm 4\sqrt{1160}}{32}$$

$$= \frac{-1 \pm 8\sqrt{290}}{32} = \frac{-1 \pm \sqrt{290}}{4}$$

$$= 3.25735$$

8. (9 pts) A rancher with 750 feet of fencing wants to enclose a rectangular area and then divide it into four pens with fencing parallel to one side of the rectangle as shown below.

$$5x + 2y = 750 \qquad y = \frac{750 - 5x}{2}$$

$$2y = 750 - 5x$$

$$A = xy$$

A. Find a function that models the total area $A(x)$ of the four pens as a function of x.

$$A(x) = x\left(\frac{750 - 5x}{2}\right) = \boxed{\frac{750x - 5x^2}{2} = 375x - 2.5x^2}$$
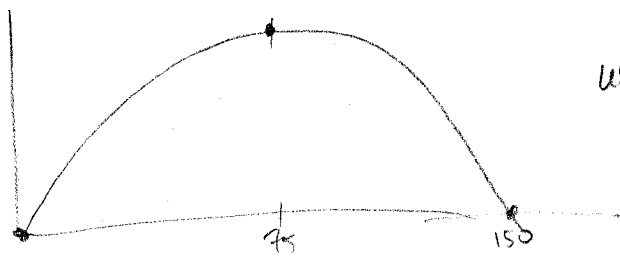
PARABOLA, OPENS DOWN

B. What is the value of x that gives the maximum area?

$$x = \frac{-B}{2A} = \frac{-375}{2(-2.5)} = \frac{-375}{-5} = 75 \qquad \boxed{x = 75 \, FT}$$

C. What is the maximum area? (don't forget to use correct units)

$$A(75) = \frac{750(75) - 5(75)^2}{2} = \boxed{14062.5 \, FT^2}$$

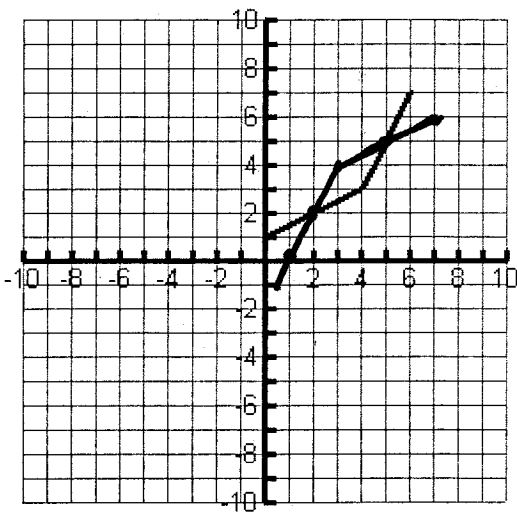USING CALCULATOR. MAX: $(75, 14062.5)$

The graph of a function is given.

4 A. Find the domain and range of *f*.   D: $[0, 6]$ R: $[1, 7]$

3 B. Sketch the graph of f^{-1}.

3 C. Find the average rate of change between *x = 2* and *x = 6*.

$$Ave = \frac{f(6) - f(2)}{6 - 2}$$

$$= \frac{7 - 2}{4} = \frac{5}{4} = \boxed{\frac{5}{4}}$$